EXPERIMENTAL INVESTIGATION OF HEAT TRANSFER IN AXISYMMETRIC VOLUMES FOR BOUNDARY CONDITIONS OF THE SECOND KIND

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Free-convection heat transfer is investigated in spherical volumes and in a horizontal cylinder with a constant heat flux density at the surface. Relations are obtained for the temperature distribution and the rate of heat transfer at the wall – liquid interface.

Studies of motion and heat transfer in natural convection are of interest from both the theoretical and practical points of view. The theoretical interest lies primarily in such features of the process as the interaction of the boundary layer with the bulk of the liquid, characterized by certain velocity and temperature distributions.

One of the most important practical applications of the problems of internal convection is related to the problem of the protracted storage of cryogenic liquids.

Convection in horizontal and vertical layers of liquid has been adequately studied for various boundary conditions. Numerical, experimental, and theoretical studies have led to data on the laws of heat transfer in layers, the nature of the motion of the liquid, and the temperature distributions [1-8]. Heat transfer in axisymmetric volumes (sphere, torus, horizontal cylinder) during the development of convection is complicated largely as a result of the specific configuration of the heat-releasing surface, and has not been investigated except for certain regularities related to the overall heat-transfer characteristics [9-13]. What has been said applies primarily to the practically important case when the heat flux density at the boundary of the volume is specified.

The present work is a continuation of the studies reported in [13, 14] and seeks to determine the characteristic convection times in a spherical volume with a given heat flux density at the surface, and to study the pattern of motion of the liquid and the reasons for the temperature stratification of the liquid over the height of the volume.

The flow was made visible by introducing powdered aluminum into a horizontal cylinder ($L/R = 1.4$) with transparent bases filled with polyethylsiloxane liquid No. 2 ($Pr = 77.5$, $\nu = 8.23 \cdot 10^{-6}$ m^2/sec).

The photographs in Fig. 1 show the characteristic features of the development of the motion and the pattern corresponding to the establishment of the flow in a horizontal circular cylinder.

Experiment shows that the pattern of motion is plane and symmetric with respect to the vertical plane passing through the axis of the cylinder. Three characteristic flows of the liquid can be distinguished in a cross section: an upward jet, flow along the heat-releasing surface, and a counter flow in the bulk of the liquid. The last two flows form a circulation loop.

Motion of the liquid along the heat-releasing surface, i.e., the flow determining the hydrodynamic boundary layer, is observed from values of the coordinate angle $\varphi \geq 15$-$20°$. In the boundary layer, against the background of the general mass motion of the medium, appear individual jets rising from the shell and spreading to the sides. On the whole the flow of the liquid here is of the nature of a traveling wave, as is particularly clearly shown in the $15 \leq \varphi \leq 90°$ region. This characteristic of the motion can be clearly seen in Fig. 1c and d.

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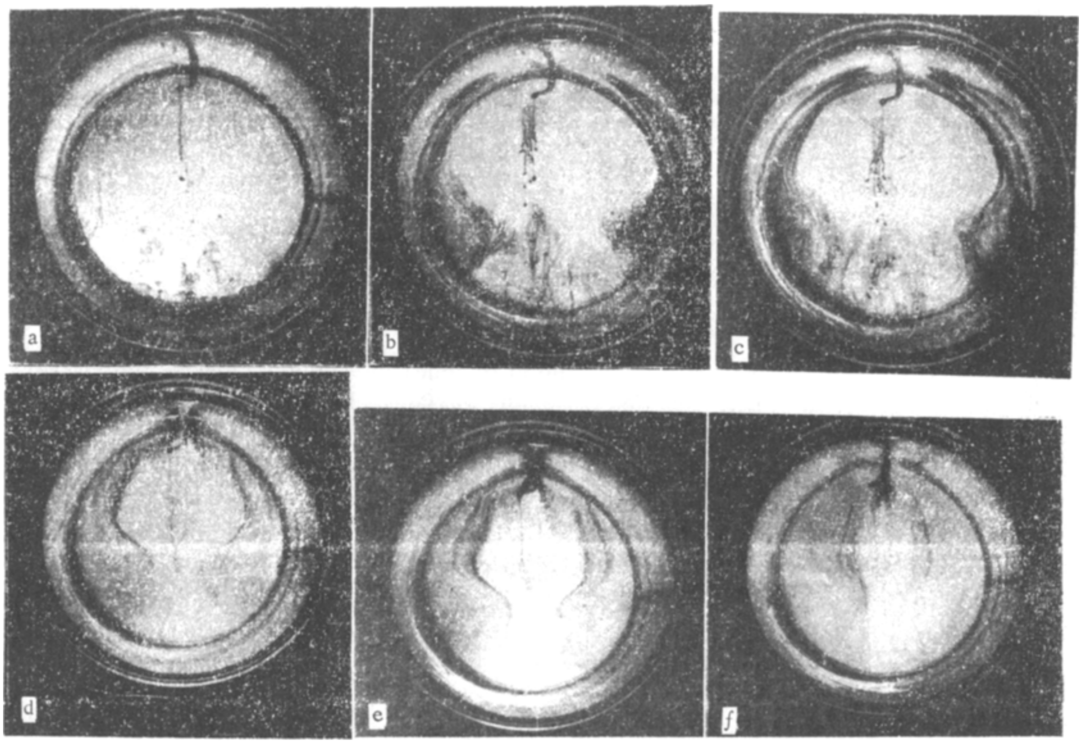

Fig. 1. Pattern of motion of a liquid in a horizontal cylinder: a) τ = 1.5 min; b) 13.5; c) 14; d) 17; e) 18.5; f) 25.

The maximum velocity of the liquid in the boundary layer ($\sim 10^{-2}$ m / sec) occurs in the region of the boundary where φ = 70–90°.

Higher along the shell the wave character disappears, the flow becomes steadier and takes on the character of a creeping motion.

Damping of the flow of the boundary layer in the upper part of the volume is a characteristic feature distinguishing the boundary layer in finite volumes heated from all sides from convective motion in infinite vertical layers. When the bulk temperature of the liquid increases upward in a finite volume the elements of liquid in the outer part of the boundary layer finally fall into the uniform density region; that is, they lose their excess angular momentum and energy with respect to the bulk of the liquid and become part of it. Thus the velocity diminishes and the mass of the boundary layer decreases. In such configurations as a sphere, torus, or horizontal cylinder the slowing down of the flow in the boundary layer aids not only the temperature and density gradients in the bulk of the liquid, but also the orientation of the heat releasing surface in the gravitational field for $\varphi > 135°$.

Along with the upward flow in the boundary layer there is a counter flow in the bulk of the liquid. In the formation of the circulation flow the peripheral regions of the bulk are set into motion first, and then the whole bulk acquires a downward motion. In the outer part the downward flow of the liquid tends to follow the curved container wall, but is forced toward the center by the jets rising from the bottom.

Single ascending and descending jets occupy an appreciable part of the volume. The ascending jets lie in the $0 \leq \varphi \leq 70°$ range and penetrate the bulk of the liquid to a height $z \cong R$ (Fig. 1a–c).*

As a result of the interaction of the downward flow and the jet flow the liquid in the lower part of the volume is well mixed and the heat entering this region is rather uniformly distributed.

One should expect the pattern of the motion of liquid in a spherical volume to be generally similar to that observed here.

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*This type of motion occurred throughout the whole experiment. The fact that the jets are not fixed in Fig. 1d–f is a result of the powder being carried away from the lower part of the volume.

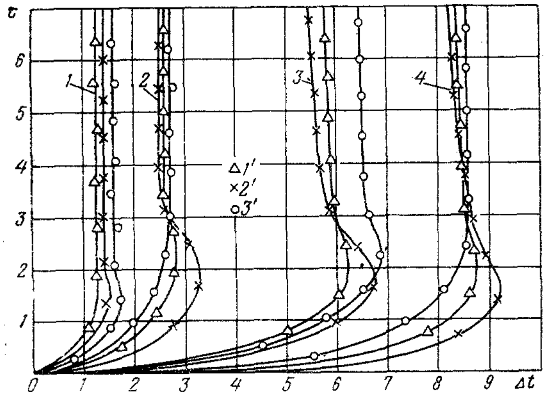

Fig. 2

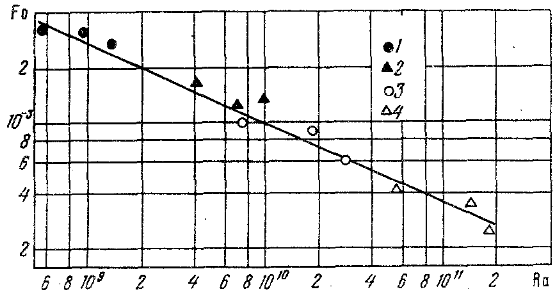

Fig. 3

Fig. 2. Change of the temperature drop between the shell and the outside of the boundary layer: 1) ethyl alcohol, diameter of vessel $\varnothing = 0.15$ m, $q = 1.06 \cdot 10^2$ W/m^2, Ra $= 1.4 \cdot 10^9$; 2) ethyl alcohol, $\varnothing = 0.3$ m, $q = 2.64 \cdot 10^2$ W/m^2, Ra $= 5.6 \cdot 10^{10}$; 3) water, $\varnothing = 0.3$ m, $q = 15.1 \cdot 10^2$ W/m^2, Ra $= 3.2 \cdot 10^{10}$; 4) ethyl alcohol, $\varnothing = 0.3$ m, $q = 13.5 \cdot 10^2$ W/m^2, Ra $= 2.8 \cdot 10^{11}$. Experimental points: 1') $\varphi = 112.5°$; 2') 135; 3') 157.5. τ is in min and Δt in °C.

Fig. 3. Limit of establishment of quasistationary heat transfer at a vertical plate [15] (solid line) and in a spherical volume (experimental points): 1) water, $\varnothing = 0.15$ m, $q = (4; 6.8; 9.4) \cdot 10^2$ W/m^2; 2) ethyl alcohol, $\varnothing = 0.15$ m, $q = (3; 5.3; 7.4) \cdot 10^2$ W/m^2; 3) water, $\varnothing = 0.3$ m, $q = (3.4; 8.4; 13.4) \cdot 10^2$ W/m^2; 4) ethyl alcohol, $\varnothing = 0.3$ m, $q = (2.6; 6.8; 12.4) \cdot 10^2$ W/m^2.

Heat transfer in a spherical volume was investigated in 0.15, 0.3, and 0.5 m diameter vessels filled with butyl and ethyl alcohol and distilled water. Heat was supplied to the surface of the vessel by an electric heater at a rate $10^2 \le q \le 10^3$ W/m^2 with $3 \cdot 10^8 \le \text{Ra} \le 10^{11}$.

We present below an analysis of the characteristic heat-transfer times in a spherical volume corresponding to nonstationary, transient, and quasistationary processes.

In the present problem the heat-transfer process is a result of the interaction of convective heat transfer by circulatory motion of the liquid and heat conduction. The time evolution of micro- and macro-transfer processes is a result of the character of the heat transfer at various stages.

Heat transfer at the wall – liquid interface is determined by processes in the boundary layer, and becomes quasistationary after the passage of the first wave of liquid along the shell. Examination of the curve showing the change of the temperature drop between the shell and the outside of the boundary layer with the time of its formation shows that the extreme points separate the process of the formation of the temperature drop into two periods (Fig. 2). The first period, corresponding to the part of the curve from $\tau = 0$ to the instant when the drop reaches its maximum, is characterized by the predominance of heat conduction in the heat-transfer process in the boundary layer. The second period is characterized by the increasing contribution of convection and ends with the establishment of a steady drop. The time to establish a steady temperature drop at the wall – liquid interface does not exceed 1.5-3 min in our experiments. Part of this time* is consumed in establishing the temperature distribution over the thickness of the shell. The brevity of the nonstationary part of the heat-transfer process at the boundary of the configuration for a negligible change in its duration from experiment to experiment, i.e., with a change in the size of the vessel, the kind of liquid, and the magnitude of the heat flux, does not permit the deduction of any relation accurately establishing the time limit of this regime.

In the range of Rayleigh numbers investigated the average time to establish quasistationary heat transfer at the wall – liquid interface corresponds to a Fourier number of $\sim 10^{-3}$.

A comparison of our experimental points corresponding to the limit of quasistationary heat transfer with a corresponding relation for a vertical plate [15] extrapolated to Rayleigh numbers greater than 10^7 shows good agreement (Fig. 3).

*Calculations performed on the assumption that at time zero all the heat supplied by the heater goes into heating the shell show that the time necessary to produce the given temperature drop is of the order of 0.1-0.5 min.

A similar conclusion follows also from very general considerations. In the initial stage of the process, when the volume of the liquid is not stratified, i.e., there is no adverse density gradient and no bulk circulatory motion, the flow of the boundary layer in a spherical volume experiences the same interaction as on a vertical plate if singularities introduced by the curvature of the shell are neglected.

The rate of heat transfer at the wall – liquid interface in the quasistationary regime can be expressed in terms of the overall heat-transfer coefficient as a function of the Rayleigh number

$$\mathrm{Nu} = C\mathrm{Ra}^k, \tag{1}$$

where $\mathrm{Nu} = \bar{\alpha}R/\lambda$; $\bar{\alpha} = \frac{1}{n}\sum_{i=1}^{n}\frac{q}{\Delta t_i(\varphi)}$; $3.5 \cdot 10^8 \leq \mathrm{Ra} \leq 10^{12}$; $C = 0.7 \pm 0.1$; $k = 0.2 \pm 0.005$.

The formation of the temperature distribution in the bulk of the liquid can be divided into three stages corresponding to nonstationary, transient, and quasistationary heat-transfer processes. During the first stage the liquid acquires a circulatory motion and the process is of a particularly nonstationary character; the rates of heating of the upper and lower regions of the volume are appreciably different – low for the lower region and high for the upper region. The nature of heat transfer in the transient regime is determined basically by the circulatory motion and nonstationary heat conduction. The heating rates at various points of the volume in this stage are leveled out and approach the value given by the mean bulk temperature law $\theta = \lambda[(t_T - t_0)/qR] = 3\mathrm{Fo}$. It may be necessary to assume a quasistationary regime characterized by the equality of the heating rates at all points of the volume to establish stationary transfer by heat conduction ($\mathrm{Fo}^* \cong 0.5$).

The time to establish the transient regime, or the upper limit of the nonstationary regime, determining the duration of the formation of the circulatory motion in the volume can be obtained by assuming that the circulatory motion is formed after a single passage of all the liquid in the volume through the boundary layer. Thus

$$\tau^* = \frac{v}{G}, \tag{2}$$

or

$$\tau^* = 0.845\frac{R^2}{\delta\bar{U}}. \tag{3}$$

Here v is the volume of the vessel; G is the volumetric flow rate through the boundary layer; $\bar{U}$ is the average velocity of the liquid in the boundary layer; and δ is the average thickness of the boundary layer.

By using Eq. (1) the relation for the thickness of the boundary layer $\delta = 2R/\mathrm{Nu}$ obtained in [13] becomes

$$\delta = 2.86\frac{R}{\mathrm{Ra}^{0.2}}, \tag{4}$$

Assuming that all the heat passing through the shell in the region of the pronounced boundary layer ($0 \leq \varphi \leq 135°$) is transferred to the bulk of the liquid in the region $0 \leq z/R \leq 1.7$, characterized by a closed heat balance, it is possible to obtain the following formula for the average velocity of the liquid in the boundary layer:

$$\frac{\bar{U}R}{a} = 0.92\mathrm{Ra}^{0.4}, \tag{5}$$

or

$$\frac{\bar{U}R}{\nu} = 0.92\frac{1}{\mathrm{Pr}}\mathrm{Ra}^{0.4}. \tag{5a}$$

Substitution of (4) and (5) into (3) gives

$$\tau^* = 0.32\frac{R^2}{a}\mathrm{Ra}^{-0.2}, \tag{6}$$

or, transforming to generalized variables

$$\mathrm{Fo}^* = 0.32\mathrm{Ra}^{-0.2}. \tag{7}$$

TABLE 1. Values of the Coefficients in Eqs. (8) and (11) Describing the Temperature Distribution in a Liquid

Liquid					
$\frac{z_i}{R}$	B_i	$C_i \cdot 10^2$	b_i	m_i	$M_i \cdot 10^2$
0÷0,6	1,245	—0,265	0,03	—444	—0,0381
0,9	1,66	—0,187	0,02	—176	—0,281
1,15	2,2	—0,15	0,01	—97,9	—0,690
1,4	4,15	0,13	—0,01	38,0	3,06
1,7	15,6	5,0	—0,05	270	1,09
1,85	27,44	12,9	—0,065	350	1,16
2,0	55,4	38,0	—0,085	437	1,32

A similar relation can be obtained also from an analysis of the heating curves for various points of a spherical volume since the onset of the transient heat-transfer regime manifests itself in a change in the curvature of these curves.

The times characteristic for various Rayleigh numbers calculated by using Eq. (7) are rather close to the results obtained in [17] where numerical methods were used to investigate natural convection in a flat region with the admission of heat through the lateral surface.

The quasistationary regime of heat transfer was not reached in our experiments since the duration of the experiments was limited by the boiling of the liquid.

In the time interval studied ($0 < \mathrm{Fo} \leq 3 \cdot 10^{-2}$) relations were obtained in generalized variables describing the time dependence of the temperature at various points on the vertical axis of a spherical volume† and in the meridional cross section on the shell.

During nonstationary heating ($0 < \mathrm{Fo} \leq \mathrm{Fo}^*$)

$$\theta_{im} = \varkappa_i [1 - \exp(-m_i \mathrm{Fo})], \tag{8}$$

where m_i and $\varkappa_i$ are dimensionless coefficients depending on the Rayleigh number and determined from Eqs. (9) and (10), respectively,

$$\frac{\exp(-m_i \mathrm{Fo}^*) - 1}{m_i} = \frac{C_i}{B_i} + \mathrm{Fo}^*, \tag{9}$$

$$\varkappa_i = B_i \mathrm{Ra}^{b_i} \left(\frac{C_i}{B_i} + \frac{1}{m_i} + \mathrm{Fo}^* \right) = B_i \mathrm{Ra}^{b_i} M_i. \tag{10}$$

In the transient regime ($\mathrm{Fo}^* \leq \mathrm{Fo} \leq 3 \cdot 10^{-2}$)

$$\theta_i = (C_i + B_i \mathrm{Fo}) \mathrm{Ra}^{b_i}. \tag{11}$$

The values of the coefficients in Eqs. (8) and (11) are listed in Tables 1 and 2. The accuracy of the calculation of the temperature using the relations given is ±10% for the lower region ($0 \leq z/R \leq 1.5$), and ±15% for the upper region ($1.5 \leq z/R \leq 2.0$).

Figure 4 shows typical curves for the temperature distribution along the vertical axis of the vessel. The character of the curves indicates large temperature gradients in the upper part of the volume and appreciably more heating than in the average bulk of the liquid in this region. These features of the temperature distribution are accounted for in most of the convection studies known at the present time by assuming the formation of a heated stratified zone as a result of the energy yielded by the boundary layer [18-21]. It must be assumed that this hypothesis represents a real process only when bounded regions are considered with heat input through the lateral walls. For heat input in all directions the heating and temperature stratification of the liquid in the upper part of the configuration are due mainly to heat flow through the upper regions of the boundary. This is tested by an energy analysis comparing the actual increments of the mean temperatures of various horizontal layers with the values of the temperature increment calculated by assuming that all the heat supplied by the heater at the boundary of a given layer goes to increase its temperature. The experimental value of the increment in the mean temperature θ_e during a chosen time interval in the range $0 < z/R \leq 0.6$ is 1.7-2.5 times smaller than that calculated under the above

†The temperature distribution along the vertical axis is sufficient to describe the temperature distribution in the bulk of the liquid since as the volume is heated the temperature is constant at each instant in each horizontal cross section.

TABLE 2. Values of the Coefficients in Eq. (11) Describing the Temperature Distribution in a Shell in the Transient Heating Regime

Shell							
φ^o	B_i	$C_i \cdot 10^2$	b_i	φ^o	B_i	$C_i \cdot 10^2$	b_i
0÷45	9,35	3,98	—0,058	135	29,6	18,5	—0,08
67,5	10,77	4,6	—0,06	157,5	43,7	29,34	—0,085
112,5	16,5	9,44	—0,07	180	55,4	38,0	—0,085

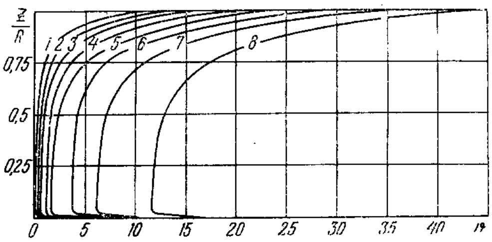

Fig. 4. Temperature distribution along the vertical axis of a spherical volume: 1) τ = 3 min, Fo = $0.65 \cdot 10^3$; 2) 5 and $1.08 \cdot 10^3$, respectively; 3) 7 and $1.52 \cdot 10^3$; 4) 11 and $2.39 \cdot 10^3$; 5) 15 and $3.25 \cdot 10^3$; 6) 25 and $5.9 \cdot 10^3$; 7) 40 and $8.66 \cdot 10^3$; 8) 68 and $15.7 \cdot 10^3$. ϑ is in degrees.

assumption. θ_e is the experimental value of the increment in the mean temperature of the layer and θ_c is the calculated value. A similar comparison for thicker layers $0 < z/R \le 0.9$, $0 < z/R \le 1.15$, and $0 < z/R \le 1.7$ shows that as the layer thickness is increased the difference between the amount of heat passing through the shell and that remaining in the layer decreases and approaches zero for the range $0 < z/R \le 1.7$. A comparison of the values of θ_c and θ_e for the layers $0.9 < z/R \le 1.15$, $1.15 < z/R \le 1.4$, and $1.4 < z/R \le 1.7$ indicates an influx of heat into these layers from below. In the upper region $1.7 < z/R \le 2.0$ there is a transfer of heat from a higher layer $1.85 < Z/R \le 2.0$ into the layer $1.7 < z/R \le 1.85$ located below. In the region $1.7 < z/R \le 2.0$ there is a heat balance to within the experimental error.

Thus in a spherical volume there are two regions characterized by a closed heat balance. The mean temperatures of these regions during the heating of the volume vary with the total heat flux entering the liquid through the surfaces of the shell bounding these regions. The difference in the heating rates in these regions is due to the difference in the ratio of the area of the shell bounding the region to the volume occupied by the given region. Since this ratio is larger for the upper region the rate of growth of the temperature is also larger in the upper region.

On the basis of the experiments performed we propose the following approximate physical model of the heat-transfer process in axisymmetric volumes with the heat flux density specified on the surface:

1. The flow of liquid in the volume is axisymmetric and is also quasistatic after the time $Fo^* = 0.32\, Ra^{-0.2}$.

2. There are dynamic and thermal boundary layers on the boundary with a heat-emitting surface.

3. In the liquid outside the boundary layer it is possible to distinguish a region of uniform temperature ($0 < z/R \le 1.7$) and a stratified zone ($1.7 < z/R \le 2.0$). The bulk region can be treated as a lumped parameter system, assuming in the first approximation that the time dependence of the bulk temperature is given by the law for the mean bulk temperature $\theta = 3Fo$. The temperature of the stratified region is a function of height and time, and can be found by a one-dimensional formulation of the equation of conservation of energy indicating a balance between the heat entering from the shell and that transmitted by conduction and convection. The contribution of convection is estimated by using the heat-transfer coefficient determined, for example, from (1).

NOTATION

a is the thermal diffusivity;
ν is the kinematic viscosity;
R is the radius;
L is the length;
z is the vertical coordinate (z = 0 for the lower point);
τ is the time;
q is the heat flux density;

φ	is the coordinate angle in a meridional cross section formed by the vertical axis and the moving radius ($\varphi = 0$ for the lower point);
Δt	is the temperature drop in the boundary layer;
$\vartheta = t_T - t_0$	is the temperature increment;
$\theta = \lambda[(t_T - t_0)/qR]$	is the temperature increment in dimensionless form;
$Pr = \nu/a$	is the Prandtl number;
$Fo = a\tau/R^2$	is the Fourier number;
$Ra = g\beta qR^4/\nu a\lambda$	is the Rayleigh number;
$Nu = qR/\Delta t\lambda$	is the Nusselt number.

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